Simulation of Wave-Current Interaction Using Novel, Coupled Non-Phase and Phase Resolving Wave and Current Models

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LONG-TERM GOALS

The long term goals of this project are to be able to predict nearshore waves, currents, and sediment transport accurately from >20m water depth through to the shoreline. We would like to accomplish this over as large an area as possible; on the order of tens of km^2, and to resolve all individual waves. Time periods simulated would be of order hours to days at maximum. We would also like to be able to directly couple these phase-resolving models with non-phase resolving models for integration into larger scale dynamics.

OBJECTIVES

The specific objectives of this project, which began 18 months ago, are to (1) Develop and test novel, fundamentally rotational phase-resolving wave-current systems which may have arbitrary order; (2) Code these theoretical systems and develop them into phase-resolving nearshore surf zone models; and (3) Couple with large scale wave/circulation models.

APPROACH

All of the budget for this project has gone to fund PhD student Yao Zhang, who is the major worker. Advisor and PI Dr. Andrew Kennedy is also working on this project. We are also working closely with other researchers who have separate funding on this topic.

Our fundamental technical approach is to represent nearshore water wave systems by retaining Boussinesq scaling assumptions, but without any assumption of irrotationality. We continue to assume a polynomial variation in horizontal velocity

$$\mathbf{u}(x,y,z;t)=\sum_{j=0}^{N}\tilde{\mathbf{U}}_n(x,y;t)f_n(q) \tag{1}$$

where $\boldsymbol{u}$ is the horizontal velocity, $f_n(q)$ is a polynomial function of $q\equiv(z+h)/(h+\eta)$, and $\tilde{\boldsymbol{U}}_n$ are coefficients that vary in horizontal coordinates and time. The specification of N, which controls the order of approximation, and f_n, which allows for asymptotic rearrangement, determines the system

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properties once the velocity expansion is integrated into Boussinesq-scaled continuity and Navier-Stokes equations. This is a generalization of the Boussinesq approach that allows for much more freedom in determining the system properties.

The resulting systems can have two forms: a classic Boussinesq-like appearance with mixed space-time derivatives but with several coupled equations; or a scaled pressure-Poisson-like form with polynomial vertical variation. Each has advantages for certain cases. We note that even though we are considering water wave systems, the scaled pressure-Poisson form may be quite useful for weakly nonhydrostatic ocean models.

WORK COMPLETED

Significant work has been accomplished by student Yao Zhang during this project. We have developed scalings, and derived both general systems and variants for specific levels of approximation (e.g. $O(\mu^2)$, $O(\mu^4)$). We have also determined analytical properties for systems up to quite high levels of approximation, and optimized properties using asymptotic rearrangement. This rearrangement can improve properties greatly, and because of our very general approach we are able to simultaneously optimize both linear dispersion and shoaling, and also nonlinear properties in some instances. We have coded and tested these equations for one horizontal dimension, and found good agreement with experimental data. We have derived and tested a new absorbing-generating sponge layer that is both accurate and much simpler and more efficient to implement than internal wavemakers (Chawla and Kirby, 2000). We have derived and partially tested an eddy viscosity-based breaking model with fewer ad hoc assumptions than other Boussinesq breaking models.

RESULTS

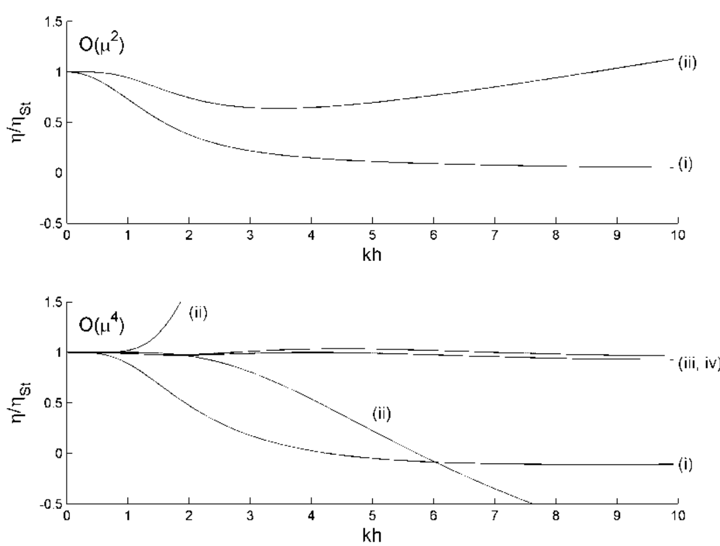

Figure 1. Nonlinear second order Stokes characteristics at lower order $O(\mu 2)$, and higher order $O(\mu 4)$, and asymptotic rearrangements to improve performance for larger wavenumbers (deeper water).

The end results of this short research project are rotational Boussinesq systems with excellent accuracy in physical properties, both linear and nonlinear. Research codes have been developed for both O(μ^2) and O(μ^4) systems in 1D, with wave breaking and bottom friction. The systems have demonstrated good accuracy for standard tests, with good efficiency in computation. Figure 1 shows analytic measures of nonlinear accuracy for different asymptotic rearrangements. To our knowledge, these rearrangements leading to increased nonlinear accuracy are unique to this project.

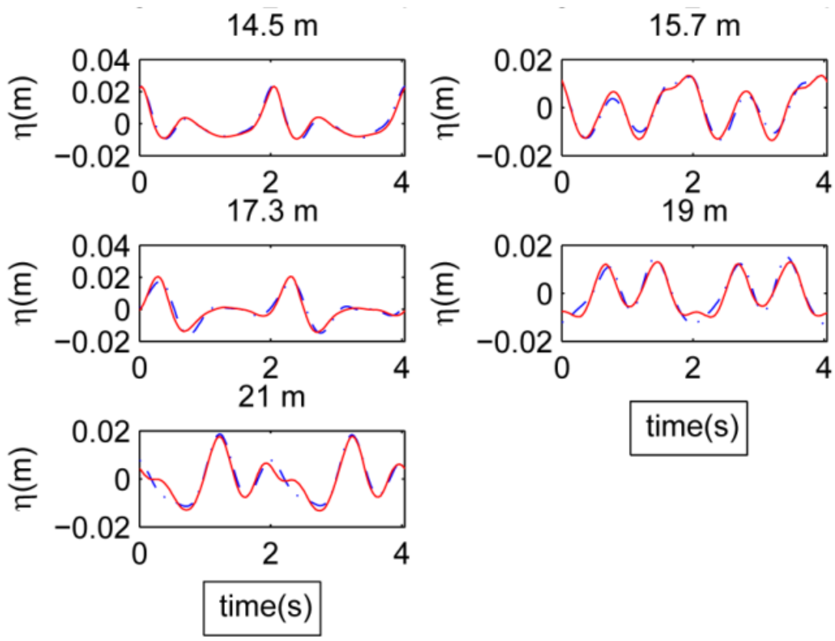

Figure 2. Time series comparisons for O(μ^4) nonlinear, non-breaking wave transformation over a shallow shoal.

Figure 2 shows comparisons between measured and computed time series for wave transformation over a submerged shoal (Dingemans, 1994). Results are very good even after the shoal, which has been a problem area for existing Boussinesq models. However, our new system has significantly more accurate dispersion, shoaling and nonlinear properties, and achieves good accuracy in both phase and amplitude of the nonlinear decomposed harmonics. One of the most significant advances of the past year has been the development of a new generating-absorbing layer that not only generates waves more accurately than existing methods, but is also easier to implement on new systems. This is an adaptation of the sponge layer concept of Israeli and Orszag (1981) to include a desired wave forcing that is a solution to the homogeneous system. For a system with variables $\mathbf{a}(\mathbf{x},t)$, it may be written as

$$[\mathbf{A}_1]\frac{\partial[\mathbf{a}]}{\partial t} + \text{ other terms } = \omega_1(\mathbf{x})[\mathbf{A}_1]\left[\mathbf{a}_{\mathbf{imp}} - \mathbf{a}\right] \tag{1}$$

where $\omega_1(\mathbf{x})$ is a non-negative, spatially-varying sponge layer coefficient, and $\mathbf{a_{imp}}$ is the desired wave solution. It is easily seen that, if $\mathbf{a}_{imp}(\mathbf{x},t)$ is a solution to the undamped system (ω_1=0), then it is also a solution to the damped system of equation (1) and passes through without change. However, if $\mathbf{a} \neq \mathbf{a}_{imp}$, then the damped system gradually is forced to the desired solution over space and time. Variations from this desired system, which include reflected waves, are damped out providing a combined absorbing-generating layer on the boundaries of the simulation. To generate a desired wave train, all that is required is a knowledge of the surface elevations and velocities for the desired free wave. These may be either linear or nonlinear, steady or unsteady. This is straightforward for many systems and is much easier to implement than internal generators like Chawla and Kirby (2000), which require Green's function solutions and have never been generalized to nonlinear generation. Because the present generation system has the feedback term ($\mathbf{a}$-$\mathbf{a_{imp}}$), it is a closed loop type of system which corrects for errors in discretization rather than the open loop systems (no feedback) in use at present. The technique may be easily transferred to different systems, and is simple to program, and is very accurate. Theoretical generation coefficients for various sponge layer integrated strengths and lengths using Nwogu's (1993) Boussinesq equations show that, given five or more water depths for the generation layer, very accurate results may be obtained. Numerical tests confirm this.

The accurate generation of nonlinear irregular waves is the most difficult task for phase-resolving models. There are two difficulties: (1) knowing details of the nonlinear target wave; and (2) generating the target waves. The present work does not help so much with (1), but can generate almost any nonbreaking target it is given. Figure 3 shows a highly nonlinear example where the nonlinear target is actually known from a large-scale computation over x=0-47m. Results from this larger domain are then used as the imposed solution for a smaller domain computation in x=17-47m, with a generation zone of 17-27m. At location A out of the sponge layer at x=35m, nonlinear results from the larger scale computation (first generation) are compared with results generated using the smaller domain (second generation). The results match almost exactly, which is an extremely difficult test, and one that no other Boussinesq-type system has ever achieved. This means that, for example, computations with one model may take place over one domain and their results used to initiate computations with another model over an overlapping domain. This will be quite important for forcing phase-resolving nearshore models, which presently tend to begin computations in water deep enough that the systems are almost linear.

A journal paper based on these findings is almost complete and is targeted to be submitted within the next month.

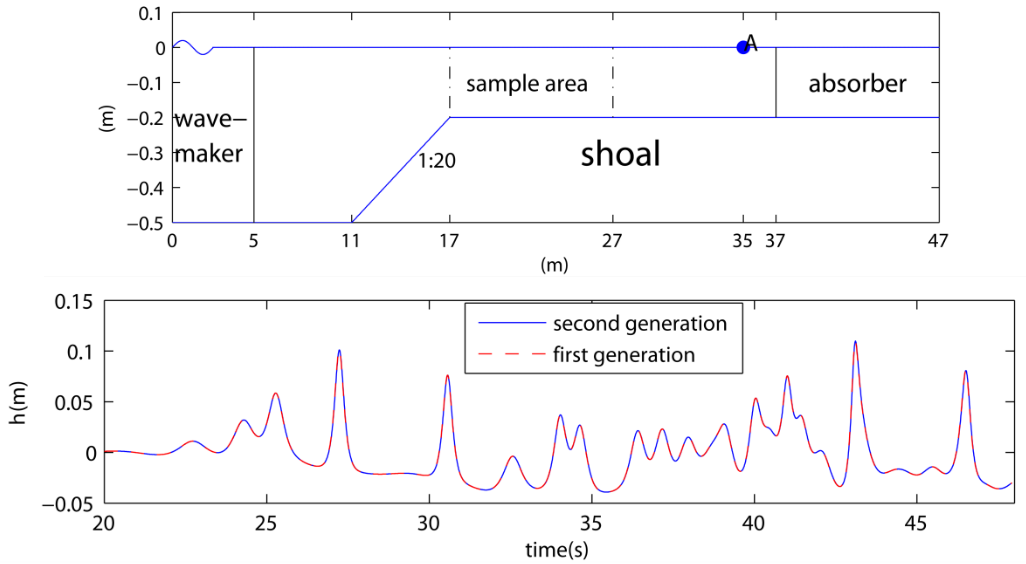

Figure 1. Time series at point A (x=35m) for nonlinear, irregular waves generated between x=17 and 27m using new technique (second generation), compared to nonlinear target solution (first generation).

The past six months have seen concerted efforts in establishing wave breaking into the inviscid models. After all, this is what is expected to be the major advantage of this type of system: a Boussinesq-scaled wave model that also represents well rotational processes. This has been accomplished using a quite simplified eddy viscosity technique, keeping only low order terms according to the shallow water scaling. Comparisons and further development are ongoing, including undertow profiles that have not been well represented by existing phase-resolving nearshore models.

Figure 4 shows preliminary comparisons with experimental measurements of wave breaking on a shoal. Agreement is quite reasonable and shows good agreement in both amplitude and phase for most stations, but shows slightly more error than inviscid computations in the same region (not shown). This is expected as breaking waves are larger amplitude, and breaking processes themselves introduce additional complexity.

The systems of the previous section are all based on mixed space-time derivatives representations of pressure, as are all classical Boussinesq forms. However, for two horizontal dimensions, simultaneous solution of two mixed space-time horizontal velocity equations is required, which increases costs considerably. An alternative methodology is to construct scaled pressure Poisson equations, which can incorporate many of the same asymptotic rearrangement techniques as was done with mixed space-time derivatives. This not only reduces the number of variables in the elliptic equations, but also allows for the possibility of turning hydrostatic ocean models into weakly nonhydrostatic models using simple changes to the pressure forcing.

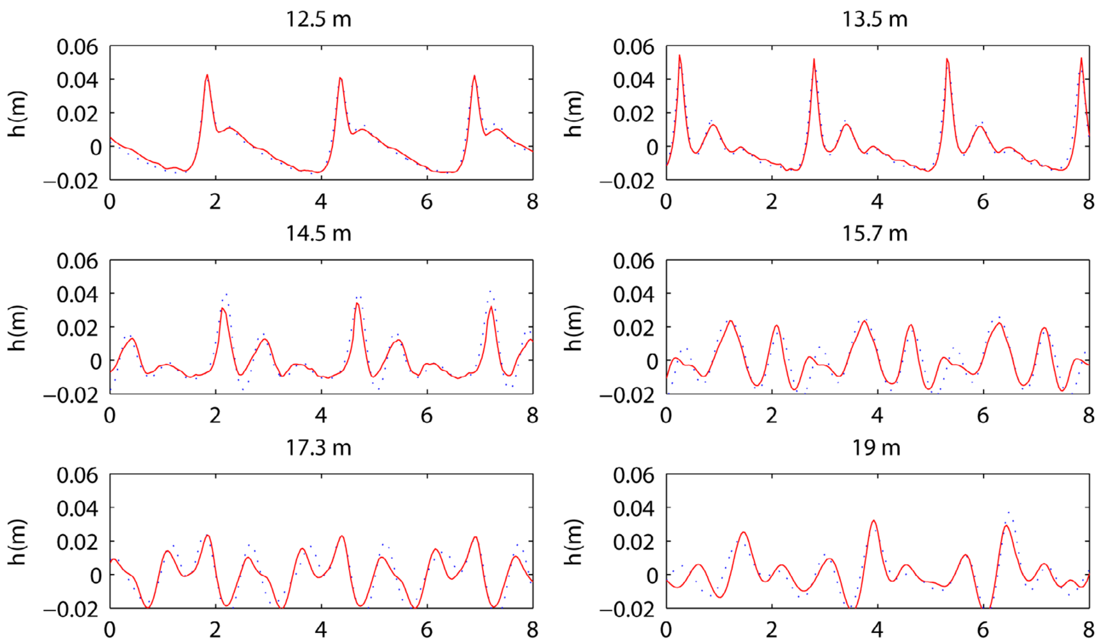

Figure 4. Preliminary computations (solid) and measurements (dashed) of breaking wave transformation over a shoal.

Another PhD student Aaron Donahue has been working on these variants with PI Andrew Kennedy (but not under ONR funding), and has accomplished significant theoretical derivations and asymptotic rearrangements to improve properties for various orders. He is now working on implementing these numerically, which is proving challenging as the simplest implementations have been showing balancing issues in regions of high slope that are reminiscent of those experienced in some finite volume hydrostatic models. Aaron is now working on solutions to these problems, and once these are overcome is expected to make rapid progress.

IMPACT/APPLICATIONS

The systems developed and tested here form a bridge between existing moderate accuracy, irrotational Boussinesq systems that can be used to simulate waves and large scale currents over relatively large coastal areas, but can not give details of hydrodynamics in the surf zone; and highly accurate Navier-Stokes models that can give excellent results over small areas but can not be used for large regions. We expect them to be particularly useful for studies in the surf zone, including sediment transport and depth-varying undertow where standard Boussinesq models do not perform well. However, we do note that obtaining the full impact from these systems will require continued development, as we have only been working on these topics for two years.

RELATED PROJECTS

This project is directly tied to NSF project 1025519, which is a collaboration between Notre Dame, the University of Texas, and the Ohio State University. The present project has funding for a PhD student, Yao, Zhang, to work on these topics in collaboration with the other workers.

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PUBLICATIONS

One journal paper has received final acceptance in Coastal Engineering, and will likely be on the web very shortly after this report is received. We have also had two conference publications arising from this project. Another journal submission is expected in the near future, dealing with generating-absorbing sponge layers, with others on the horizon.

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